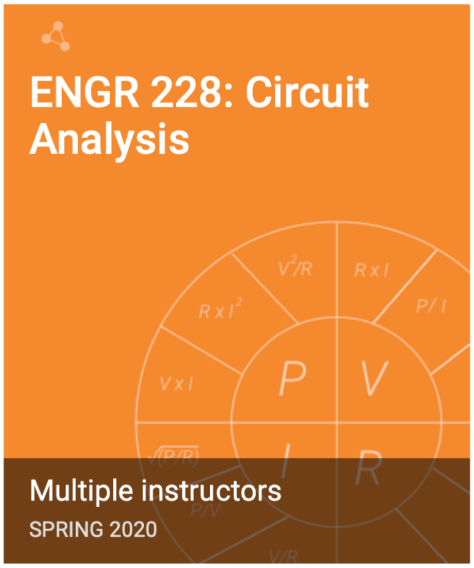


Chapter 3.2

Analysis Techniques

Node Voltage Method

Engr228 - Circuit Analysis

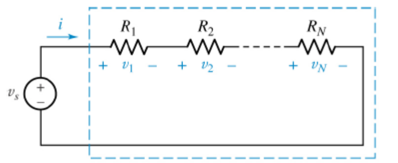
Spring 2020

Dr Curtis Nelson

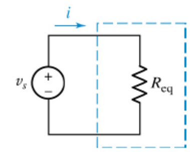
Section 3.2 Objective

- Learn to apply the node-voltage method to analyze an electric circuit of any configuration, so long as it is linear and planar.

Resistors in Series



(a)



(b)

$$v_s = v_1 + v_2 + \cdots + v_N$$

$$v_s = R_1 i + R_2 i + \cdots + R_N i$$

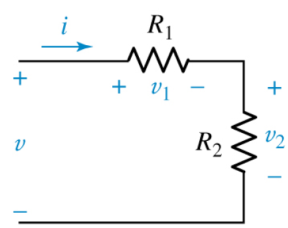
$$= (R_1 + R_2 + \cdots + R_N)i$$

$$v_s = R_{eq} i$$

$$R_{eq} = R_1 + R_2 + \cdots + R_N$$

Voltage Division

Resistors in series “share” the voltage applied to them.



$$i = \frac{v}{R_1 + R_2}$$

$$v_2 = i R_2 = \left(\frac{v}{R_1 + R_2}\right) R_2$$

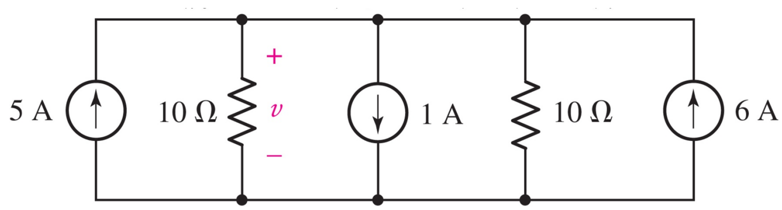
$$v_2 = \frac{R_2}{R_1 + R_2} v$$

$$v = v_1 + v_2$$

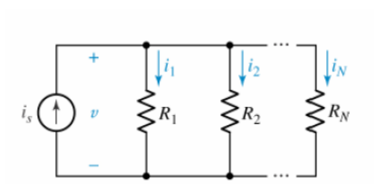
$$= i(R_1 + R_2)$$

Parallel Connections

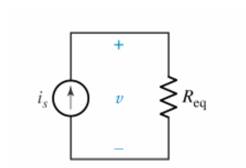
- Elements in a circuit connected head-to-head and tail-to-tail have a common voltage across them and are said to be connected in ***parallel.***



Resistors in Parallel



(a)



(b)

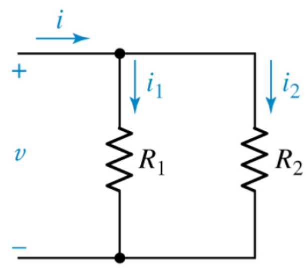
$$i_s = i_1 + i_2 + \cdots + i_N$$

$$i_S = \frac{v}{R_1} + \frac{v}{R_2} + \cdots + \frac{v}{R_N} = \frac{v}{R_{eq}}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}$$

Two Resistors in Parallel

$$R_{eq} = R_1 \| R_2$$
$$= \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

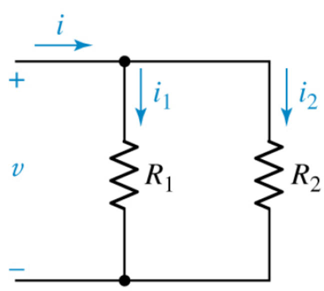


$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

Connecting resistors in parallel makes the equivalent resistance ***smaller. Always.***

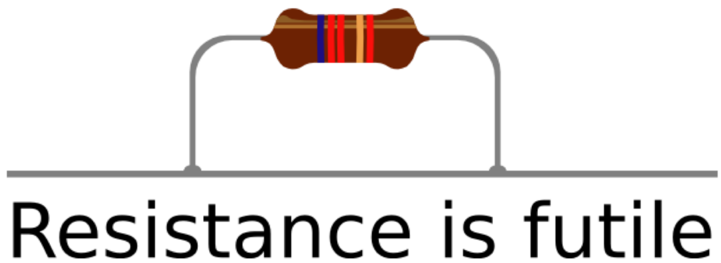
Current Division

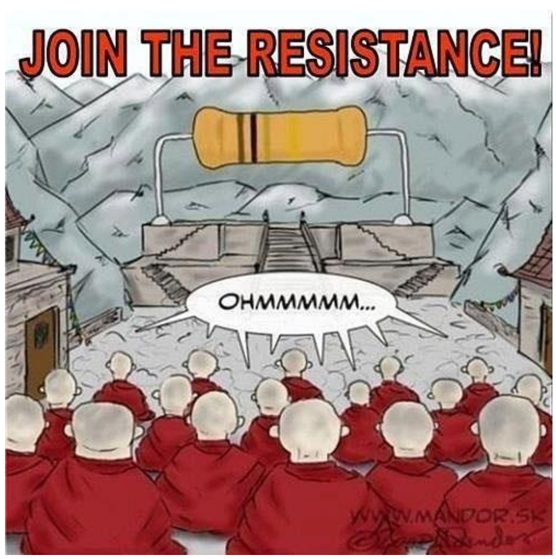
Resistors in parallel "share" the current through them.



$$i_2 = \frac{v}{R_2}$$
$$= \frac{i(R_1 \| R_2)}{R_2}$$
$$= \frac{i}{R_2} \frac{R_1 R_2}{R_1 + R_2}$$

$$i_2 = i \frac{R_1}{R_1 + R_2}$$

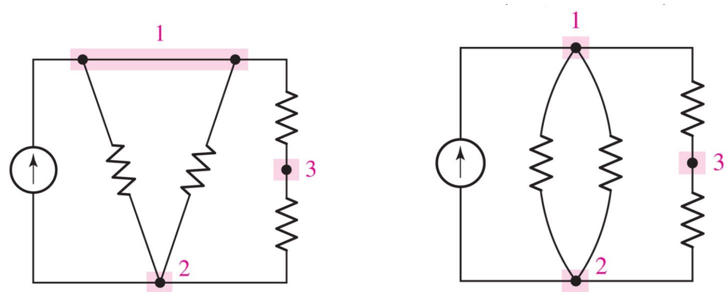

Resistance is futile


JOIN THE RESISTANCE!
OHMMMMM...
WWW.MANDOR.SK

Circuit Analysis

- We need an organized method of applying KVL, KCL, and Ohm's law;
- *Nodal* analysis assigns *voltages* to each node, and then we apply *Kirchhoff's Current Law* to solve for the *node voltages;*
- *Mesh* analysis assigns *currents* to each mesh, and then we apply *Kirchhoff's Voltage Law* to solve for the *mesh currents.*

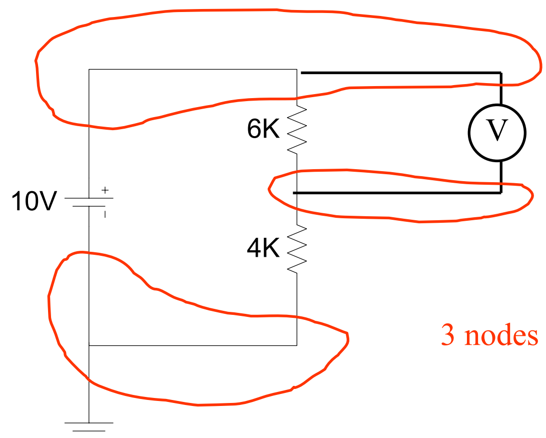
Review - Nodes, Paths, Loops, Branches



- These two circuits are equivalent.
- There are three *nodes* and five *branches:*
  - *Node*: a point at which two or more elements have a common connection;
  - *Path*: a sequence of nodes;
  - *Branch*: a single path in a circuit composed of one simple element and the node at each end of that element;
  - *Loop*: a closed path.

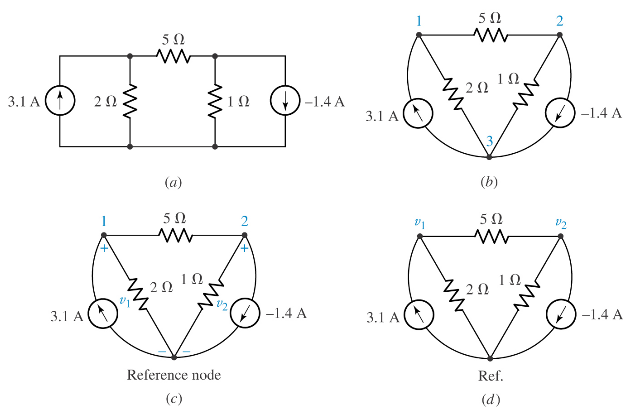
Node Example

- Node = every point along the same wire



Nodes

- How many nodes in the circuits below?



Notes on Writing Nodal Equations

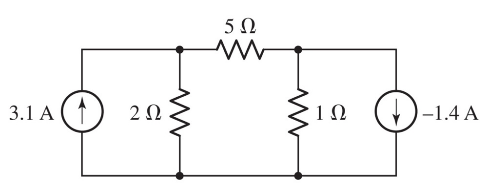
- All terms in the equations are in units of ***current;***
- Everyone has their own style of writing nodal equations
  - The important thing is that you remain consistent.
- Probably the easiest method if you are just getting started is to remember that:

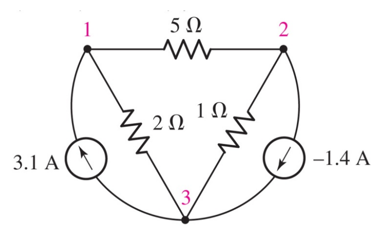
current entering a node = current leaving the node

- Current directions can be assigned arbitrarily, unless they are previously specified.

The Nodal Analysis Method

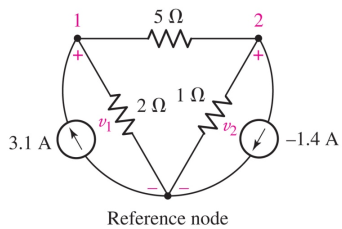
- Assign voltages to every node relative to a reference node.

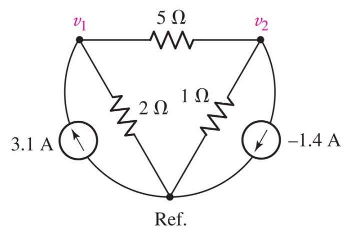




Choosing the Reference Node

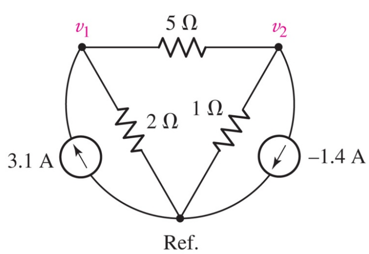
- By convention, the bottom node is often the reference node;
- If a ground connection is shown, then that becomes the reference node;
- Otherwise, choose a node with many connections;
- Assign the reference node a value of 0.00 volts.





Apply KCL to Find Voltages

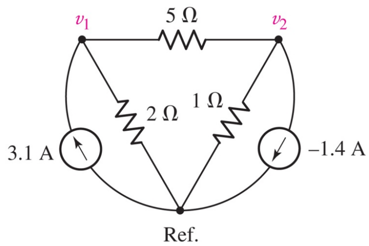
- Assume reference voltage = 0.00 volts;
- Assign current names and directions;
- Apply KCL to node v_1 (Σ out = Σ in);
- Apply Ohm's law to each resistor.



$$\frac{v_1}{2} + \frac{v_1 - v_2}{5} = 3.1$$

Apply KCL to Find Voltages

- Apply KCL to node v_2 (Σ out = Σ in);
- Apply Ohm's law to each resistor.



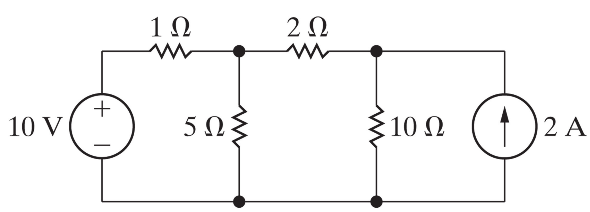
$$\frac{v_1 - v_2}{5} = \frac{v_2 - 0}{1} + (-1.4)$$

We now have two equations for the two unknowns v_1 and v_2 and we can solve them simultaneously:

$v_1 = 5V$ *and* $v_2 = 2V$

Example Problem Page 96 (Nilsson 11th)

Solve for the node voltages in the circuit below.



Answer: Left node = 9.09 V
Right node = 10.91 V

Nodal Analysis: Dependent Source Example

Determine the power supplied by the dependent source.

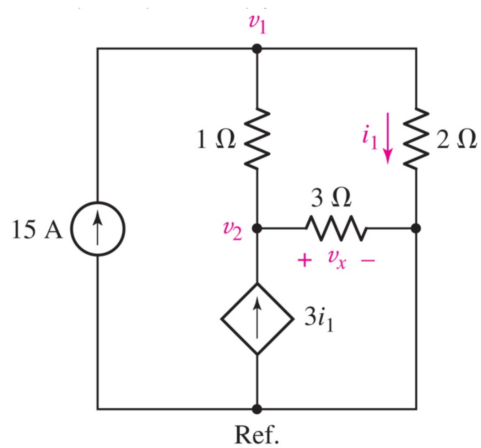
Key step: eliminate i_1 from the equations using $v_1 = 2i_1$

$$15 = \frac{v_1 - v_2}{1} + \frac{v_1 - 0}{2}$$

$$\frac{v_1 - v_2}{1} + 3i_1 = \frac{v_2 - 0}{3}$$

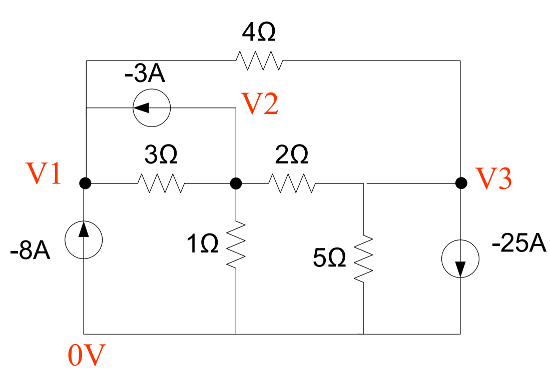
$$i_1 = \frac{v_1 - 0}{2}$$

Answer: 4.5 kW being generated

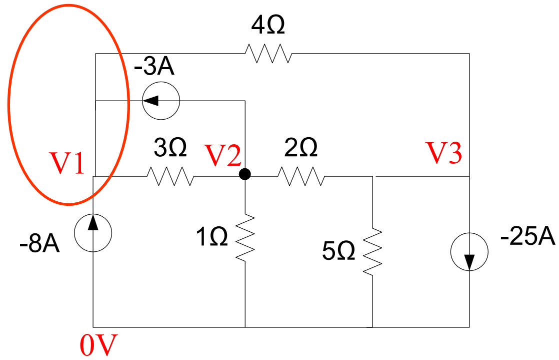


Node Voltage Example

- How many nodes are in this circuit?
- How many nodal equations must you write to solve for the unknown voltages?



Node Voltage Example – Node V1



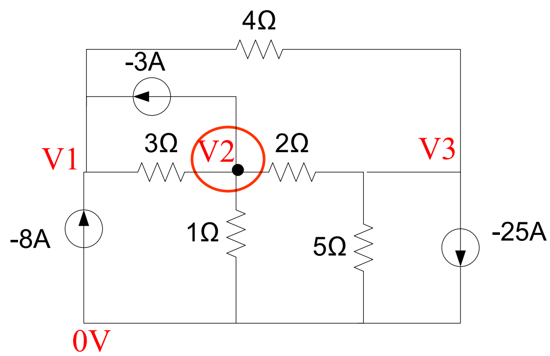
At node V_1

$$8+\frac{V1-V3}{4}+3+\frac{V1-V2}{3}=0$$

$$96+3V1-3V3+36+4V1-4V2=0$$

$$7V1-4V2-3V3=-132$$

Node Voltage Example – Node V2



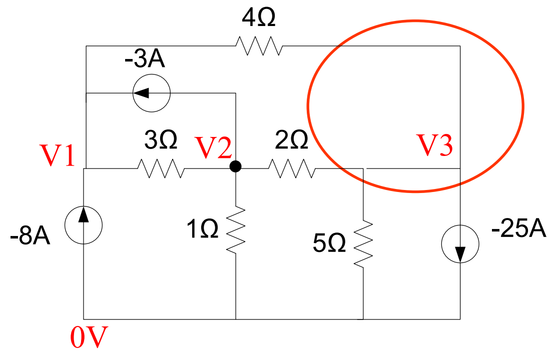
At node V_2

$$\frac{V2-V1}{3}-3+\frac{V2-V3}{2}+\frac{V2-0}{1}=0$$

$$2V2-2V1-18+3V2-3V3+6V2=0$$

$$-2V1+11V2-3V3=18$$

Node Voltage Example – Node V3



At node V_3

$$\frac{V3-V2}{2}+\frac{V3-V1}{4}-25+\frac{V3-0}{5}=0$$

$$10V3-10V2+5V3-5V1-500+4V3=0$$

$$-5V1-10V2+19V3=500$$

Node Voltage Example – Solution

$$7V1-4V2-3V3=-132$$

$$-2V1+11V2-3V3=18$$

$$-5V1-10V2+19V3=500$$

Answer: *V1 = 0.956V*

V2 = 10.576V

V3 = 32.132V

Section 3.2 Summary

- You learned to apply the node-voltage method to analyze an electric circuit of any configuration, so long as it is linear and planar.